

## HIGH TEMPERATURE THERMAL PROPERTIES OF MAGNESIUM SILICIDE INVESTIGATED BY MOLECULAR DYNAMICS SIMULATION

Meena Rittiruum<sup>a,b,\*</sup>, Keerati Maneesai<sup>c</sup>, Korakot Matarat<sup>a,d</sup>, Athorn Vora-ud<sup>b</sup>,  
Sirakan Yokhasing<sup>a,b</sup>

<sup>a</sup>Simulation Research Laboratory, Center of Excellence on Alternative Energy, Research and Development Institution, Sakon Nakhon Rajabhat University,

Sakon Nakhon, 47000, Thailand

<sup>b</sup>Program of Physics, Faculty of Science and Technology, Sakon Nakhon Rajabhat University, Sakon Nakhon, 47000, Thailand

<sup>c</sup>Faculty of Science and Engineering, Kasetsart University Chalermphrakiat Sakonnakhon Province Campus, Sakon Nakhon, 47000, Thailand

<sup>d</sup>Program of Computer, Faculty of Science and Technology, Sakon Nakhon Rajabhat University, Sakon Nakhon, 47000, Thailand

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### ABSTRACT

The thermal properties are important for processing conversation heat energy to electricity of Magnesium Silicide ( $\text{Mg}_2\text{Si}$ ) thermoelectric material. This work, we investigate the thermal properties of  $\text{Mg}_2\text{Si}$  by using molecular dynamics (MD) simulation. The cluster atomic model was designed by using  $\text{Mg}_{512}\text{Si}_{256}$  based on  $\text{CaF}_2$  structure for MD calculation. The Morse-type and Busing-Ida type were performed for inter-atomic interaction based on MXDORTO. Our calculated results composed lattice parameter, compressibility, linear thermal expansion coefficient, energy, heat capacity, and thermal conductivity, respectively, they present at versus temperature 300 – 1200 K. In addition, the heat flux auto-correlation and thermal conductivity depend on time correlation was presented. The calculated imply that the thermal conductivity of  $\text{Mg}_2\text{Si}$  become to  $\sim 1 \text{ W m}^{-1} \text{ K}^{-1}$  at high temperature.

KEYWORDS: Thermal conductivity; Green-Kubo relation; Heat capacity; MXDORTO

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Corresponding authors; e-mail: meena@snru.ac.th, Tel&Fax: +664-274-4319

### INTRODUCTION

Magnesium silicide is a non-toxicity thermoelectric (TE) material [1]. General, it exhibits n-type TE material. Whereas, the Bi-, Sb- and Al-doping lead to p-type TE material [2]. In addition, the  $\text{Mg}_2\text{Si}$  is intermediated thermoelectric power generator at high temperature applications with maximum operating temperature of around 600 K [3]. So far, the structural behaviors of  $\text{Mg}_2\text{Si}$  have been investigated by energy dispersive synchrotron X-ray diffraction. It was found that, the crystal structure of  $\text{Mg}_2\text{Si}$  is anti-fluorite ( $\text{CaF}_2$ ) type (Pearson symbol;  $cF12$ , space group symbol;  $Fm\bar{3}m$  and space group number; 225) with lattice parameter of 0.635 nm and density of  $1.90 \text{ g cm}^{-3}$  [4–6]. As well know, the  $\text{Mg}_2\text{Si}$  exhibits low thermal conductivity at room to middle temperature [7–9]. In addition, the experimental shows the heat

capacity at constant volume is  $67.6 \text{ J mol}^{-1} \text{ K}^{-1}$  [10], heat capacity at constant pressure is  $67.9 \text{ J mol}^{-1} \text{ K}^{-1}$  [10], and thermal conductivity  $5.2 – 10.5 \text{ W m}^{-1} \text{ K}^{-1}$  [11, 12] at room temperature, respectively. However, the thermal properties such as lattice parameter, compressibility, linear thermal expansion coefficient, internal energy, heat capacity, and thermal conductivity at high temperature is a few reported. This work, we investigated the thermal properties of  $\text{Mg}_2\text{Si}$ . This presents are help to understand behavior of TE as well as future applications at high temperature.

### MATERIALS AND METHODS

#### Computational details

The MD calculation on thermal properties of  $\text{Mg}_2\text{Si}$  was performed using the system of 768 ions that composed Mg (cations) 512 ions, and Si

**Table 1** The conditions for MD calculation of Mg<sub>2</sub>Si

Calculation	$L, B, \alpha_{lin}$	$C_p, C_v, C_d$	$\kappa$
<b>System</b>			
-Particles	768 Particles (512 cations and 256 anions)		
	Mg = 512, Si = 256		
-Structure	CaF <sub>2</sub> crystal structure		
<b>Control</b>			
-Temperature	Scaling method	Scaling method	Nose method [14]
-Pressure	Scaling method	No control method	Andersen method [15]
<b>Number of steps</b>	100,000	100,000	100,000

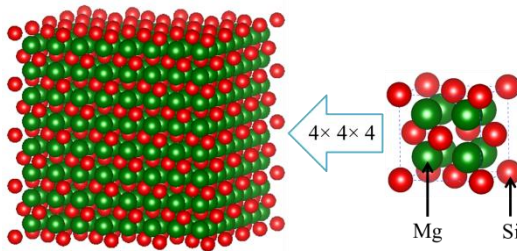
**Table 2** The inter-atomic potential parameter for Mg<sub>2</sub>Si

Ions	$z$	$a$	$b$	$c$		$D_{ij}$	$\beta_{ij}$	$r_{ij}^*$
Mg	1.2	1.926	0.11405	20	Pair Mg–Si	15	2.25	2.369
Si	-2.4	1.659	0.11405	0				

(anions) 256 ions in dimension 4×4×4 unit cells. The cluster atom model was initially arranged in the CaF<sub>2</sub> type crystal structure as shown in Fig. 1. The calculation conditions of the lattice parameter ( $L$ ), compressibility ( $B$ ), linear thermal expansion coefficient ( $\alpha_{lin}$ ), heat capacity at constant pressure ( $C_p$ ), heat capacity at constant volume ( $C_v$ ), heat capacity of lattice dilatational terms ( $C_d$ ), and thermal conductivity ( $\kappa$ ) as shown in Table 1. This calculation was performed by MXDORTO [13].

The Morse-type [16] and Busing-Ida type [17] potential function were employed for inter-atomic interaction, as show in equation. (1).

$$U_{ij}(r_{ij}) = \frac{z_i z_j e^2}{r_{ij}} + f_0 (b_i + b_j) \exp\left(\frac{a_i + a_j - r_{ij}}{b_i + b_j}\right) - \frac{c_i c_j}{r_{ij}^6} + D_{ij} \{ \exp[-2\beta_{ij}(r_{ij} - r_{ij}^*)] - 2\exp[-\beta_{ij}(r_{ij} - r_{ij}^*)] \} \quad (1)$$



**Fig. 1** The Mg<sub>512</sub>Si<sub>256</sub> cluster atoms model for Mg<sub>2</sub>Si 4×4×4 unit cells.

Here,  $f_0$  is repulsion between atoms in vacuum (4.186),  $z_i$  and  $z_j$  are the effective partial electronic charges on the  $i^{\text{th}}$  and  $j^{\text{th}}$  ions, respectively.  $r_{ij}$  is the inter-atomic distance,  $r_{ij}^*$  is the bond length of the cation-anion pair in vacuum. The parameters of  $a$ ,  $b$  and  $c$  are the characteristic parameters depending on the ion species. The potential function,  $D_{ij}$  and  $\beta_{ij}$  describe the depth and shape of this potential, respectively. The first term describes the Coulomb interactions and the second term denotes core repulsions. The third term is a Morsetype potential, and applied only to cation-anion pairs. The values of the inter-atomic potential parameters used in the present study are given in Table 2.

The calculations were performed on cluster computer, OS-Rocks Clusters 6.1.1 (based on Centos 6.5) 64 bit, total 6 cores (Intel® Core(TM) 2 Quad CPU Q9500 @ 2.83GHz 2.83 GHz) and total 8 GB of RAM.

## RESULTS AND DISCUSSION

The  $L$  of Mg<sub>2</sub>Si carried out by MD calculation at pressure 0.1 MPa and temperature 300 – 1200 K as shown in Fig. 2. Our calculated  $L$  with value of  $a = b = c = 6.3516 \pm 0.0004 \text{ \AA}$  at room temperature show good agreement with the experimental data [6, 18, 19]. It was note that, the structure of Mg<sub>2</sub>Si was expanded with increasing temperature. Our calculated of  $L$  show a few error

at high temperature and denote that high accurate of MD. In addition,  $L$  was adopted to investigate the  $B$  and  $\alpha_{\text{lin}}$  following by equations. (2) and (3);

$$B = \frac{3}{L(P_0)} \left( \frac{\partial L(P)}{\partial P} \right)_T \quad (2)$$

$$\alpha_{\text{lin}} = \frac{1}{L(T_0)} \left( \frac{L(T) - L(T_0)}{T - T_0} \right)_P \quad (3)$$

Here,  $L(P)$ ,  $P_0$ ,  $L(T)$ , and  $T_0$  are  $L$  at pressure  $P(\text{Pa})$ , atmospheric pressure, the  $L$  at  $T(\text{K})$  and room temperature, respectively. The calculated of  $B$  and  $\alpha_{\text{lin}}$  as show in Fig. 3.

Our calculated of  $B$  exhibits value  $6.2 \text{ MPa}^{-1}$  at 300 K slowly increase to  $7.65 \text{ MPa}^{-1}$  at 1200 K. Meanwhile, the  $\alpha_{\text{lin}}$  exhibits  $8.6 \mu\text{K}^{-1}$  at 350 K increase like exponential function to  $10.48 \mu\text{K}^{-1}$  at 1200 K. These results help to understand and confirm that the structure of  $\text{Mg}_2\text{Si}$  has expanded as exponential function and affect to the heat capacity and thermal conductivity.

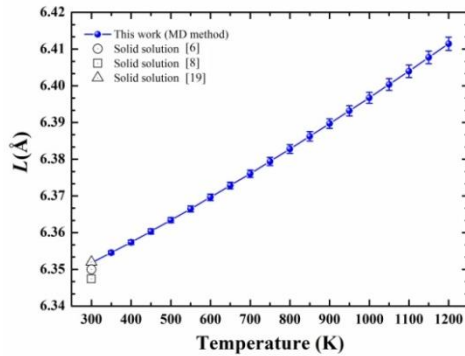


Fig. 2 Calculated of lattice parameters for  $\text{Mg}_2\text{Si}$  at various temperatures.

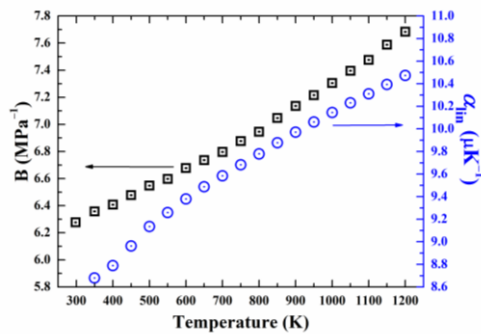


Fig. 3 Calculated of compressibility and linear thermal expansion coefficient for  $\text{Mg}_2\text{Si}$  at various temperatures.

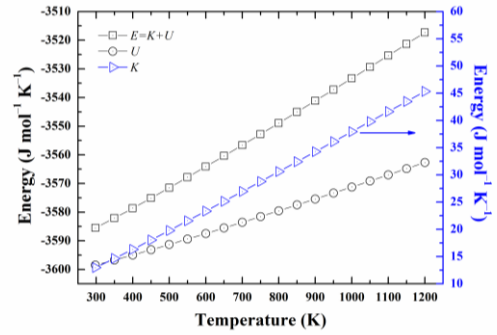


Fig. 4 Calculated of internal energy of  $\text{Mg}_2\text{Si}$  at various temperatures.

The kinetic energy ( $K$ ), potential energy ( $U$ ) and total energy ( $E$ ) were directly carried out by MD calculation can evaluate by equations. (4) – (6);

$$E = U + K \quad (4)$$

$$U = U_{\text{Coulomb}} + U_{\text{short}} \quad (5)$$

$$= U_{\text{Coulomb}} + U_{\text{vdW}} + U_{\text{srr}} + U_{\text{Morse}}$$

$$K = \frac{1}{3Nk_B} \sum_i m_i v_i^2 \quad (6)$$

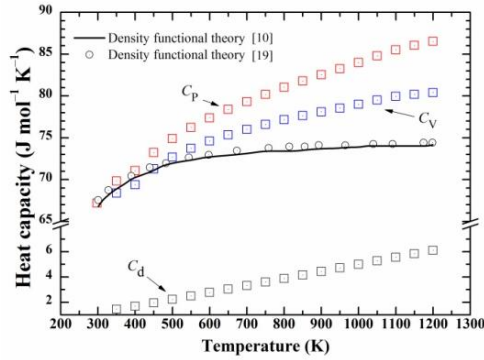
Here,  $U_{\text{Coulomb}}$ ,  $U_{\text{vdW}}$ ,  $U_{\text{srr}}$ ,  $U_{\text{Morse}}$ ,  $N$ ,  $k_B$ ,  $m_i$  and  $v_i$  are Coulomb potential, van der Waals attraction, short range repulsion, Morse potential energy, number of atom, Boltzmann constant, mass of atom  $i$  and, velocity of atom  $i$ , respectively, Our calculated of internal energy for  $\text{Mg}_2\text{Si}$  as shown in Fig. 4.

The  $C_p$  can evaluated by  $C_v + C_d$ ; where  $C_v$  regards to gradient of the total internal energy and  $C_d$  can evaluated by using  $L$ ,  $B$ , and  $\alpha_{\text{lin}}$  as following in equations. (7) – (9);

$$C_p = C_v + C_d \quad (7)$$

$$C_v = \left( \frac{\partial E(T)}{\partial T} \right)_V \quad (8)$$

$$C_d = \frac{(3\alpha_{\text{lin}})^2 V_m(T)}{B} T \quad (9)$$



**Fig. 5** Calculated of heat capacity of Mg<sub>2</sub>Si at various temperature.

Here  $V_m(T)$  is the molar volume at temperature  $T(K)$ . Our calculated of  $C_d$  show a few value about of  $1.4 \text{ J mol}^{-1} \text{ K}^{-1}$  at 300 K increased to  $6.1 \text{ J mol}^{-1} \text{ K}^{-1}$  at 1200 K. It denote that perturbation term of lattice vibration. Our calculated of  $C_v$  show agree with experimental data of Wang [10] at 350 – 500 K. It slightly increases above 500 K. The  $C_p$  and  $C_v$  were began constant above 1100 K thus agree with Dulong-Pertit law. Our calculated of  $C_p$  show agreement values with experimental data at 300 – 450 K, while more than experimental data above 450 K [19].

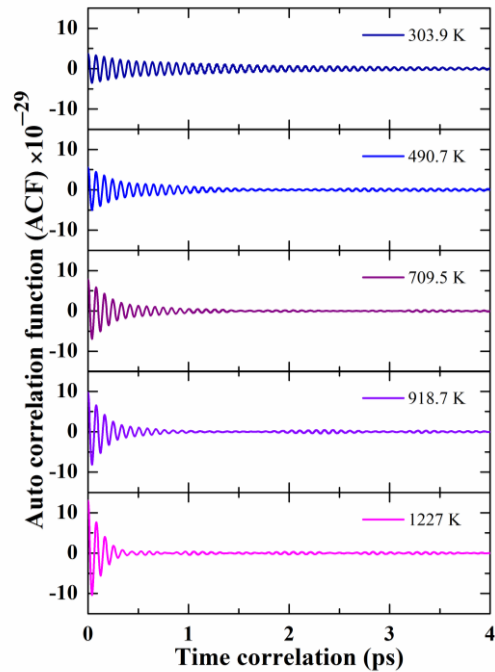
The lattice thermal conductivity can be investigated by using the Green-Kubo relation [20]. It obtained from time integral of heat flux auto-correlation function (ACF) of the energy current as following by equation. (10) – (12);

$$\kappa_{\text{lat}} = \frac{R}{3N_A T^2} [L(T)]^3 \int_0^\infty \langle S(t)S(0) \rangle dt \quad (10)$$

$$S(t) = \frac{1}{[L(T)]^3} \left[ \sum_j E_j v_j + \frac{1}{2} \sum_j \sum_{i \neq j} r_{ij} (f_{ij} v_j) \right] \quad (11)$$

$$E_j = \left\{ \frac{1}{2} m_j v_j^2 + \frac{1}{2} \sum_{i \neq j} U_{ij}(r_{ij}) \right\} - E_{\text{av}} \quad (12)$$

where  $R$ ,  $N_A$ ,  $S(t)$ ,  $t$ ,  $m_j$ ,  $v_j$ ,  $f_{ij}$ ,  $E_j$  and  $E_{\text{av}}$  are the gas constants =  $8.314 \text{ J K}^{-1} \text{ mol}^{-1}$ , Avogadro constant, the heat flux form auto-correlation function (ACF), time correlation, mass of atom  $j$ , velocity of atom  $j$ , force between atom  $i$  and  $j$ ; energy of atom  $j$  and average energy of the system, respectively.



**Fig. 6** The calculated of auto-correlation function for Mg<sub>2</sub>Si at versus time correlation.

The expectation of auto-correlation function,  $\langle S(t)S(0) \rangle$ , and calculated of  $\kappa_{\text{lat}}$  for Mg<sub>2</sub>Si as shown in Fig. 6 and 7. The ACF take approximately 4.0 ps and decay to zero, while the  $\kappa_{\text{lat}}$  has value about  $8.48 \text{ W m}^{-1} \text{ K}^{-1}$  at 4.0 ps, which occurred at at 303.9 K. It should be that, the ACF and  $\kappa_{\text{lat}}$  has been quick decay to zero where increasing temperature. In addition, the ACF is direct variation of temperature while  $\kappa_{\text{lat}}$  opposite.

The thermal conductivity of Mg<sub>2</sub>Si at versus temperature as shown in Fig. 8. Our calculated of  $\kappa_{\text{lat}}$  exhibits  $8.48 \text{ W m}^{-1} \text{ K}^{-1}$  at 303.9 K decreased to  $1.52 \text{ W m}^{-1} \text{ K}^{-1}$  at 1227 K. We using the calculation result of  $\kappa_{\text{lat}}$  compared with literature data thus composed many method such as, solid state reaction (SSR) [11, 12, 21, 22], spark plasma sintering (SPS) [23, 24], plasma activated sintering (PAS) [25, 26] and field-activated and pressure-assisted synthesis (FAPAS) [27].

All literatures exhibit the total thermal conductivity which can evaluate by  $\kappa = \kappa_{\text{lat}} + \kappa_e$ , here  $\kappa_e$  is electron thermal conductivity term. While, the MD calculation carried out the lattice thermal conductivity which

less than experimental data [11, 21, 23 – 27] and more than of experimental data [12, 22]. It indicate that good TE property, because of good TE materials are required low thermal conductivity [1, 3]. Moreover, we present the  $\kappa_{\text{lat}}$  above 900 K shown slightly decrease to  $1 \text{ W m}^{-1} \text{ K}^{-1}$ . Our calculated of thermal conductivity can be predicted by equation (13);

$$\begin{aligned} \kappa_{\text{lat}} [\text{W m}^{-1} \text{ K}^{-1}] & \\ = \kappa_0 + 23.8 \exp[-0.004T(\text{K})]; \quad \kappa_0 &= 1.53 \end{aligned} \quad (13)$$

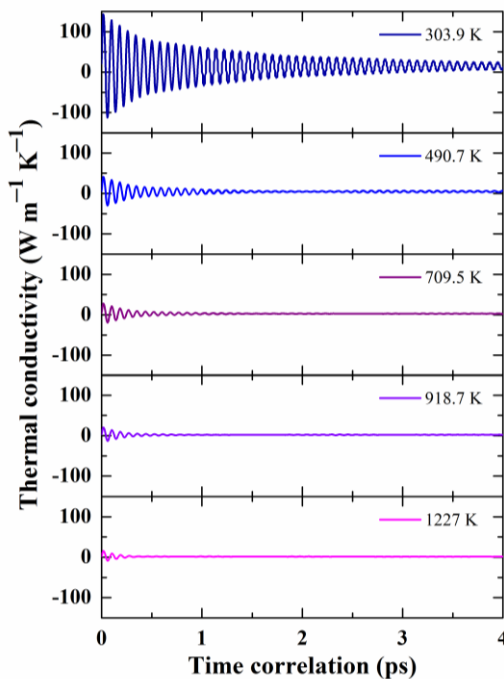


Fig. 7 The calculated of lattice thermal conductivity for Mg<sub>2</sub>Si at versus time correlation.

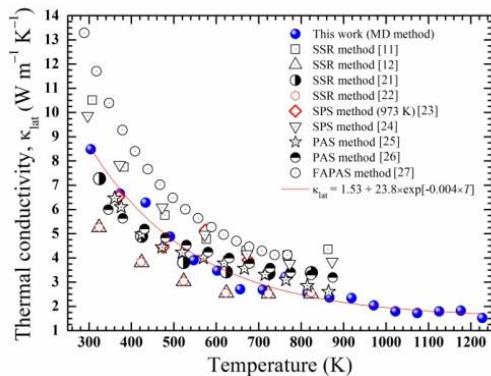


Fig. 8 The calculated of lattice thermal conductivity together with literature data at various temperature.

## CONCLUSION

The thermal properties composed  $L$ ,  $B$ ,  $\alpha_{\text{lin}}$ ,  $C_p$ ,  $C_v$ ,  $C_d$ , ACF, and  $\kappa_{\text{lat}}$  of Mg<sub>2</sub>Si has been successfully investigated by MD simulation. The  $L$ ,  $B$  and  $\alpha_{\text{lin}}$  indicated that structure of Mg<sub>2</sub>Si is expansion and contraction as exponential function as well as contribute to  $C_p$ ,  $C_v$ ,  $C_d$ , and  $\kappa_{\text{lat}}$  also change as exponential function. Our calculated of  $\kappa_{\text{lat}}$  estimated that the  $\kappa_{\text{lat}}$  decreased to  $1 \text{ W m}^{-1} \text{ K}^{-1}$  at high temperature.

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